# Computational Design of Disordered Optical Metamaterials Beyond the Quasistatic Regime

Ethan Farabee, Material Science and Engineering
Mentor: Yang Jiao, Assistant Professor
School for Engineering of Matter, Transport, and Energy

QR CODE

Research question: Can we use simulations to predict properties of metamaterials dielectric characteristics to achieve a negative reflective index?

## Abstract

**A metamaterial is a composite of types of materials in a thin layered array, metals and dielectrics.** Metamaterials interact with electromagnetic waves differently than conventional materials due to their anisotropic dispersion relation resulting in a negative index of refraction. The layered material dispersion relation changes depending on the orientation layering of the system. With data collected on the permativities of two different substances, **a model of the metamaterial can be simulated using MATLAB to predict a specific value for the dielectric constant.**

## Background

The negative index of refraction propagates electromagnetic waves in an anisotropic manner, **but the orientation and the incoming wavelength can change the permittivity of the material (Fig 1).** A dispersion relation is the display of the permittivity in both the parallel and perpendicular directions to the incident wavelength. **This research project is finding ways of simulate the dispersion relation that gives a negative index.**

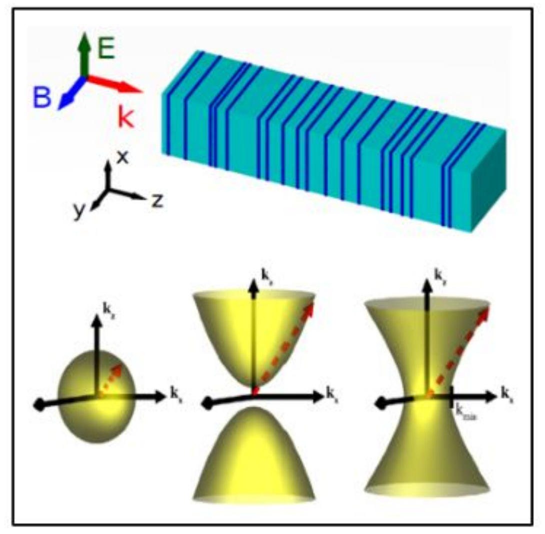


Fig 1) diagram of a metamaterial [2] (upper diagram) and the dispersion relation (lower diagram) in elliptical and hyperbolic forms

## Dispersion Relation

Material optical properties change depending on the wavelengths of the incident electromagnetic wave. **To simulate the metamaterial dispersion relation, experimental values of the wave propagation within the material (metal and dielectric) with thickness ratio of the layers can theoretically calculate the anisotropic permittivity**. Using a dataset, the index of refraction (Both real and imaginary) of the material can be calculated across the wavelength range and a function for the dielectric permittivity (Fig 2). The relationship between wavelength and permittivity, or dispersion relation, can also be shown in elliptical form (Fig 3). **For a metamaterial that has already been produced, the dispersion relation can be simulated using a simpler graphical calculation (Fig 4).** For both ideal and real systems, the same rule of mixing and anisotropic wave propagation equations can be used for both.

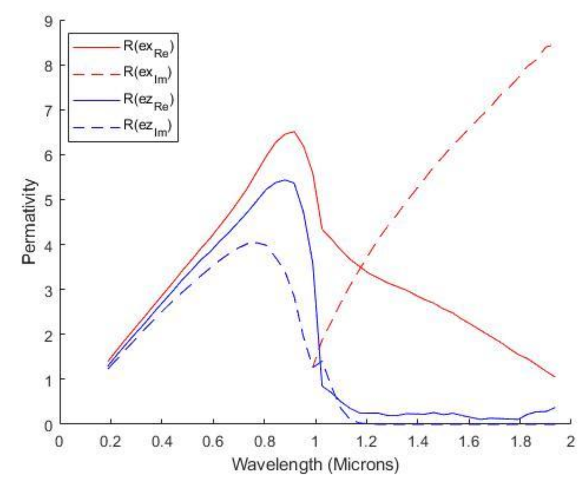


Fig 2) dielectric permittivity simulation of the TiO2 and Si layering. Both the real and the imaginary values are prevalent in the system.

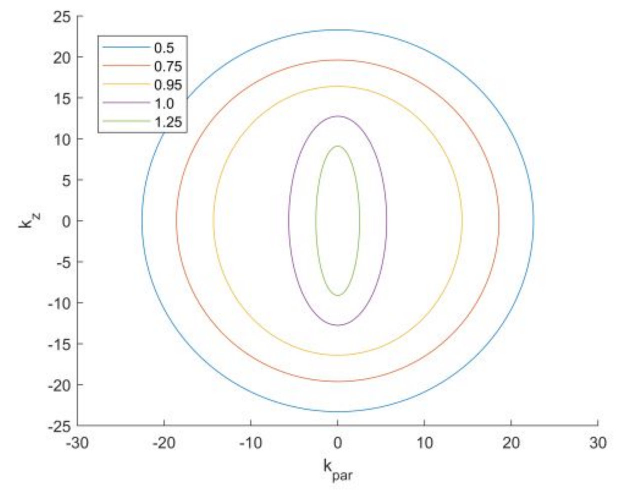


Fig 3) The dispersion relation of the TiO2 and Si system. The relation follows the elliptical system and increase propagation as you decrease wavelength.

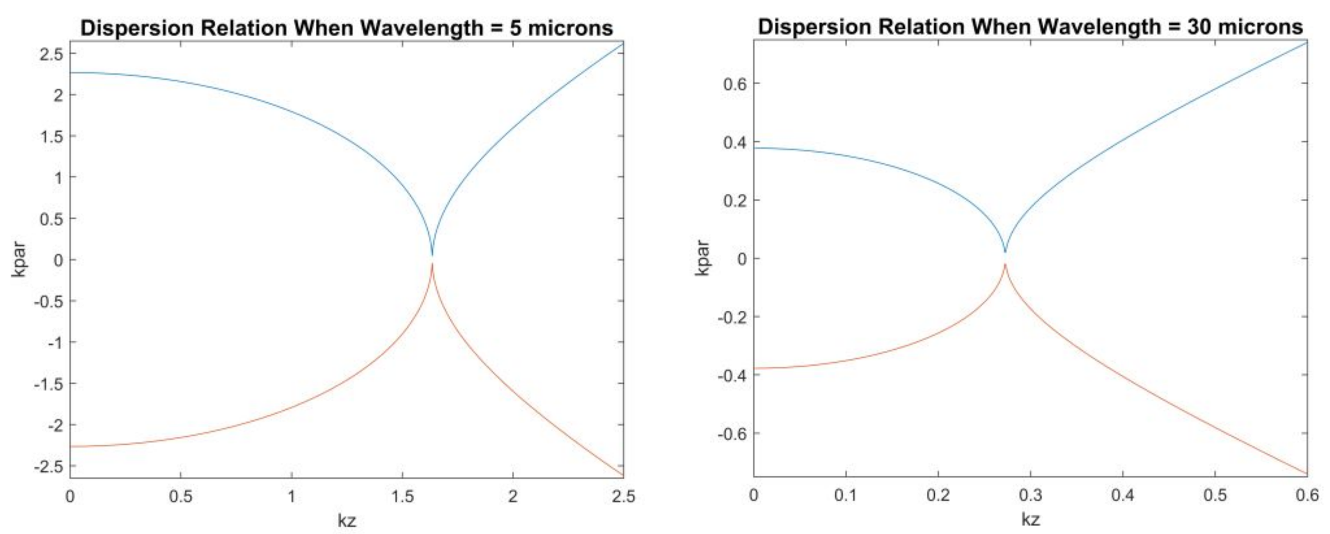


Fig 4) Dispersion relation of measured metamaterial with varying wavelengths. Shorter wavelengths (left) display a larger gap of absorption than the larger wavelength (right).

## Discussion

Dispersion relation graphs took two forms, elliptical and parabolic. The elliptical dispersion relation gives us useful information on wave propagation in an anisotropic manner, but **the parabolic dispersion relation shows that the metamaterial refraction index maintained a negative value past a given wavelength.** Not all metamaterials contain a negative refraction value so simulation of the dispersion relation can show the compatibility of the materials. **With the wave propagation in the parallel direction being dependant on the wavelength of the incident beam, the perpendicular permittivity is the majority factor when determining the dielectric permittivity (Fig 5).**

$$\frac{k_x^2}{\epsilon_z} + \frac{k_z^2}{\epsilon_x} = \frac{\omega^2}{c^2} = k_0^2.$$

Fig 5) Permittivity Equation [1] of a metamaterial with direction dependance.

With the permittivity equation, we can retrieve information on the wave propagation in both the real and imaginary index. With the calculation of the dispersion relation, future modeling of the metamaterial layering can be made with only experimental data. The modeling of the metamaterial can predict dielectric properties without having to physically form the structure.

## References

[1] Wood B, Pendry J B, Tsai D P. *Directed subwavelength imaging using a layered metal-dielectric system.* Physical Review B, 2006, 74(11): 115116.
[2] J. Kim, S. Torquato. *Effective electromagnetic wave properties of disordered stealthy hyperuniform layered media beyond the quasistatic regime*. Optica, 10 965-972 (2023).
[3] S. Yu, C. W. Qiu, Y. Chong, S. Torquato, N. Park. *Engineered disorder in photonics*. Nature Reviews Materials, 6 226 (2021).
[4] W. Shi, D. Keeney, D. Chen, Y. Jiao, S. Torquato. *Computational Design of Anisotropic Stealthy Hyperuniform Composites with Engineered Directional Scattering Properties*. Physical Review E, in press (2023)
[5] M. Landstorfer, B. Prifling, V. Schmidt. *Mesh generation for periodic 3D microstructure models and computation of effective properties*. Journal of Computational Physics, 110071 (2021)

FURI

Ira A. Fulton Schools of Engineering
Arizona State University