

# Determine failure mode and bending load capacity of carbon fiber reinforced composite square tubes for design of support structure of UAS (Unmanned Aerial System).

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## What is the load capacity of square carbon fiber tubes used as the structure of UAS (Unmanned Aerial System)?

### Methods:

**Microsoft Office Excel:** Used for compilation of estimated weights and location of components on preliminary design of the aircraft. Weights were compiled and adjusted for 3G flight loads. A lumped mass model was made, and integration was used to calculate shear and moment diagrams. The maximum bending moment was adjusted for a 1.5 safety factor, common in the aviation industry.

**MATLAB:** Used to apply tensile, compressive, and buckling formulas. Code was created to find the maximum moment supported as a function of width and thickness of flange. It was then used to find the maximum load for a matrix of width and thickness values. The data was used to graphically find the optimal sizing of a carbon fiber tube for a given bending moment.

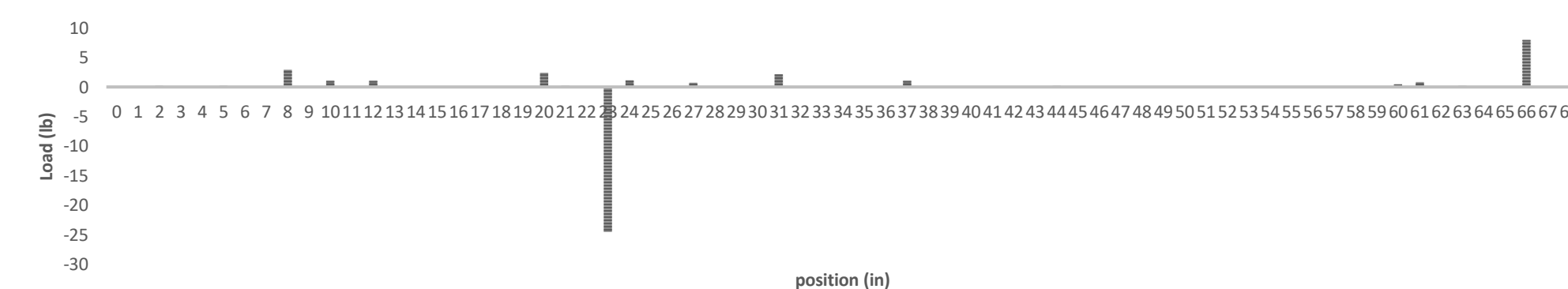
**Physical Testing:** Five test specimens were acquired and were tested until failure with pure bending moment load. Specimens were clamped to a surface and weight was hooked to the other end. Water was gradually added until failure. The weight was then measured, and results recorded.

### Assumptions:

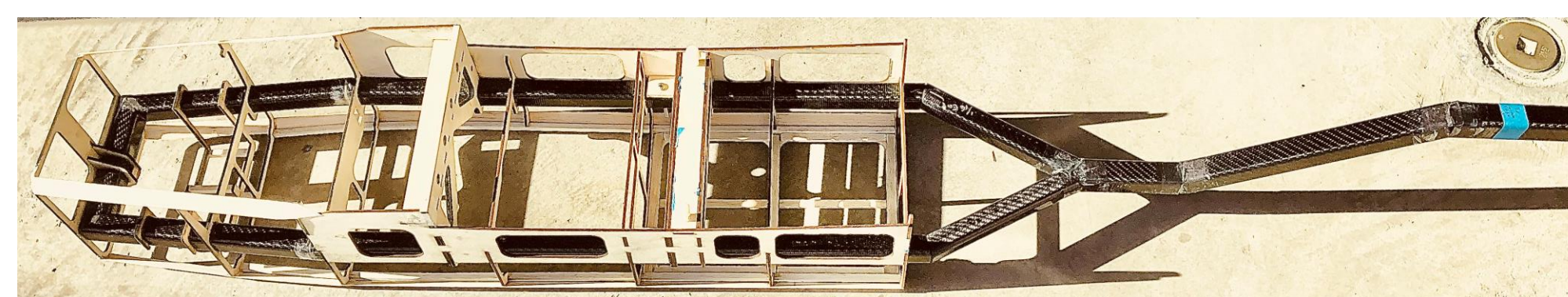
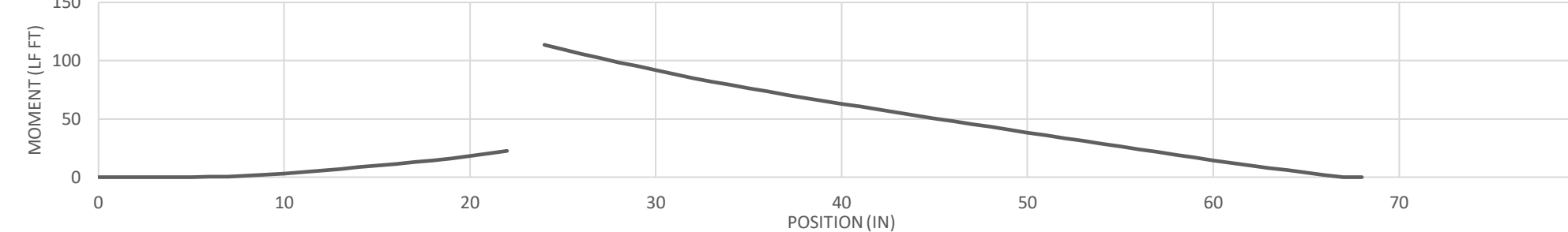
- A safety factor of 1.5 is used to ensure failure does not occur. That factor is common in the aviation industry.
- Carbon fiber usually shows a compressive strength that is much lower to its tensile strength. This is due to the brittle nature of carbon fiber and fibrous nature of the composite. Compressive strength can vary from 10% to 60% percent of tensile strength. In this case it is assumed to be 20%.
- The compressive modulus of elasticity is assumed to be 2.5 MSI. Compressive modulus of elasticity is usually less than tensile modulus of elasticity.
- In order to ease calculations and avoid use of finite element analysis software, the tensile and compressive loads are assumed to be supported by the top and bottom flanges of the square stock, respectively.
- Shear stress due to bending is calculated using only the left and right flanges of the carbon fiber stock.

### Load Calculation:

LOADS AT 1G (FIG 1)



MOMENT IN LB/FT (3G) (FIG 2)



### Failure Modes:

**Tensile Stress:** The maximum tensile stress used is the tensile yield stress of the composite. The rule of mixtures was used to calculate the total yield stress of the composite. The rule of mixtures was also used to account for carbon fiber layers in multiple directions, since carbon fiber is a non-isotropic material. The following formulas were used:

$$\sigma = \frac{My}{I} \quad \sigma_{tot} = A_{matrix}\sigma_y + A_{fiber}\sigma_y$$

**Compressive Stress:** Compressive yield stress is assumed to be 20% of the tensile yield stress of carbon fiber reinforced composites.

**Shear stress due to bending:** Shear stress caused by the bending of beam structures that occurs in the axial direction of the beam. The maximum shear stress used is the maximum shear stress of the resin. The shear stress was determined to be substantially lower than the maximum shear stress of the resin. The following formula was used:

$$\tau = \frac{3V}{2A}$$

**Buckling:** Failures that occurs due to instability caused by compressive stress on slender structures. Since one of the flanges is in compression, local panel instability occurs, causing it to buckle. A critical buckling compressive strength can be calculated using Euler's buckling theory. The formula is derived from Euler's buckling formula but applied for surfaces supported on all sides.

$$\sigma_{buckling} = \frac{k\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{a}\right)^2 \text{ where } k_a = 5.23 + 0.16\left(\frac{a}{b}\right) = 5.39$$

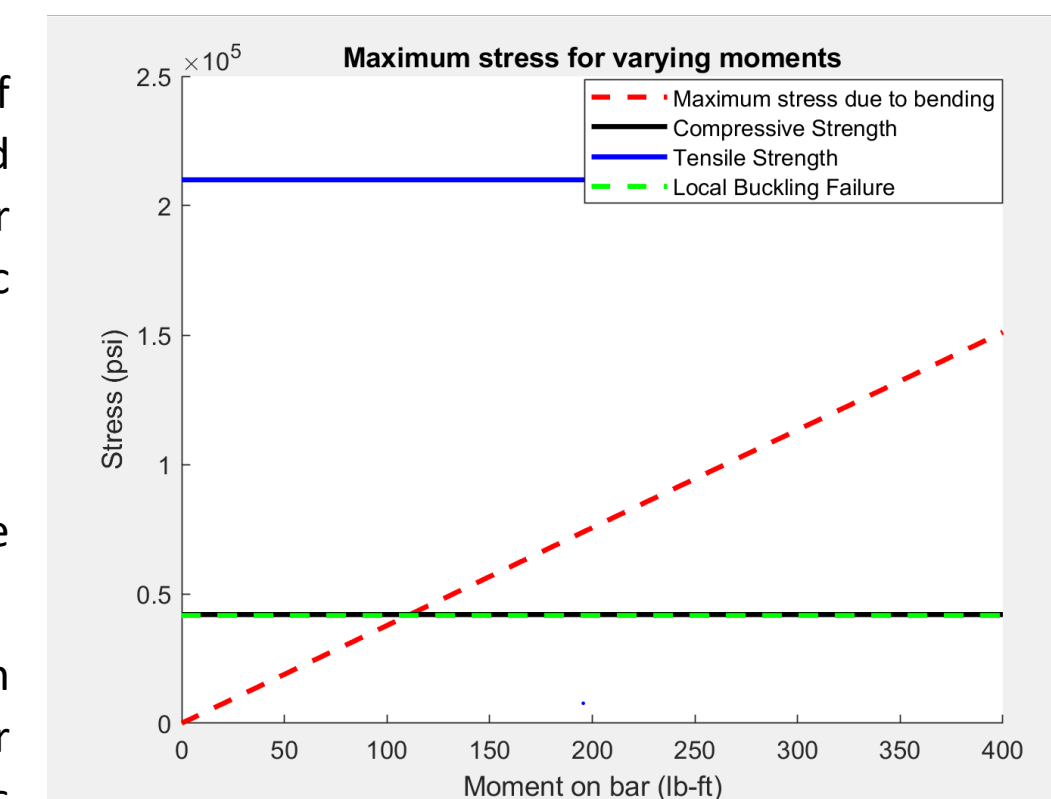
Kecman's Method is then applied using the buckling stress, compressive yield stress, and stock dimensions as input. Kecman's method focuses on relating the slenderness of a flange to the buckling stress in order to propose a series of formulas that govern the maximum moment supported. Three different cases are possible, depending on the buckling stress and yield stress. The following formulas are used:

### Testing Results:

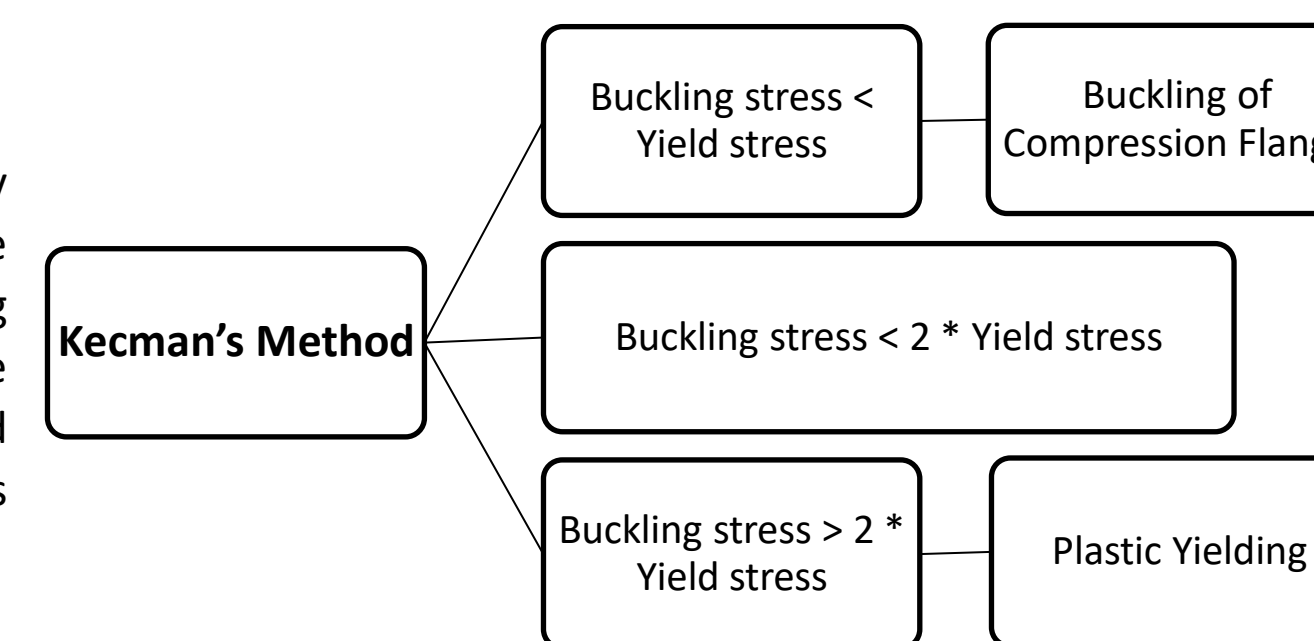
Testing Results (Fig 4)					
Manufacturer	Flange Width (in)	Flange Thickness (in)	Max Moment (lb ft)	Predicted Max Moment (lb ft)	Percent Error
Unknown	0.788	0.0385	102.52	96.54	5.83%
Rockwest	0.83	0.04	102.91	110.22	7.10%
Clearwater	1.07	0.035	133.66	130.57	2.31%
Rockwest	0.95	0.04	120.275	132.65	10.37%
Rockwest	0.83	0.04	104.4	110.22	5.57%



Testing Specimens (Fig 5)



Failure for Varying Moment (Fig 3)



$$M_{max1} = \sigma_y t x^2 * \frac{3x + 5a_e}{6x}$$

$$M_{max2} = M_{el} + (M_{pl} - M_{el}) \left( \frac{\sigma_{buckling} - \sigma_y}{\sigma_y} \right)$$

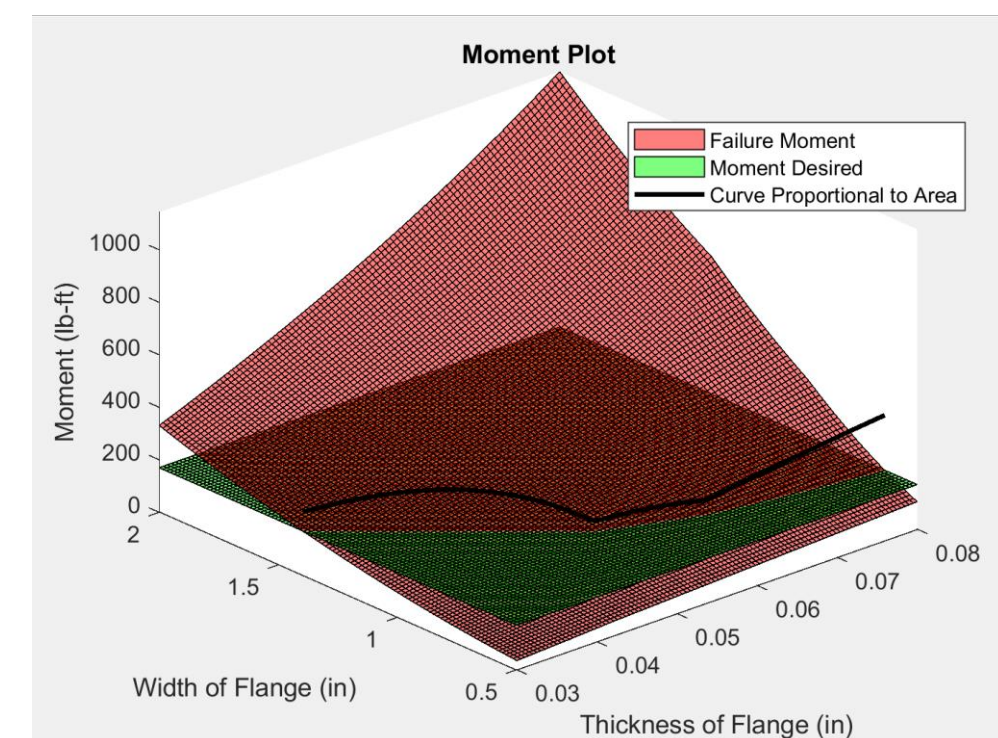
$$M_{max3} = M_{pl}$$

$$a_e = x \left( 0.7 \left( \frac{\sigma_{buckling}}{\sigma_s} \right) + 0.3 \right)$$

$$M_{pl} = \sigma_y t \left( \frac{3x^2}{2} \right) \quad M_{el} = \sigma_y t \left( \frac{4x^2}{3} \right)$$

### Data analysis:

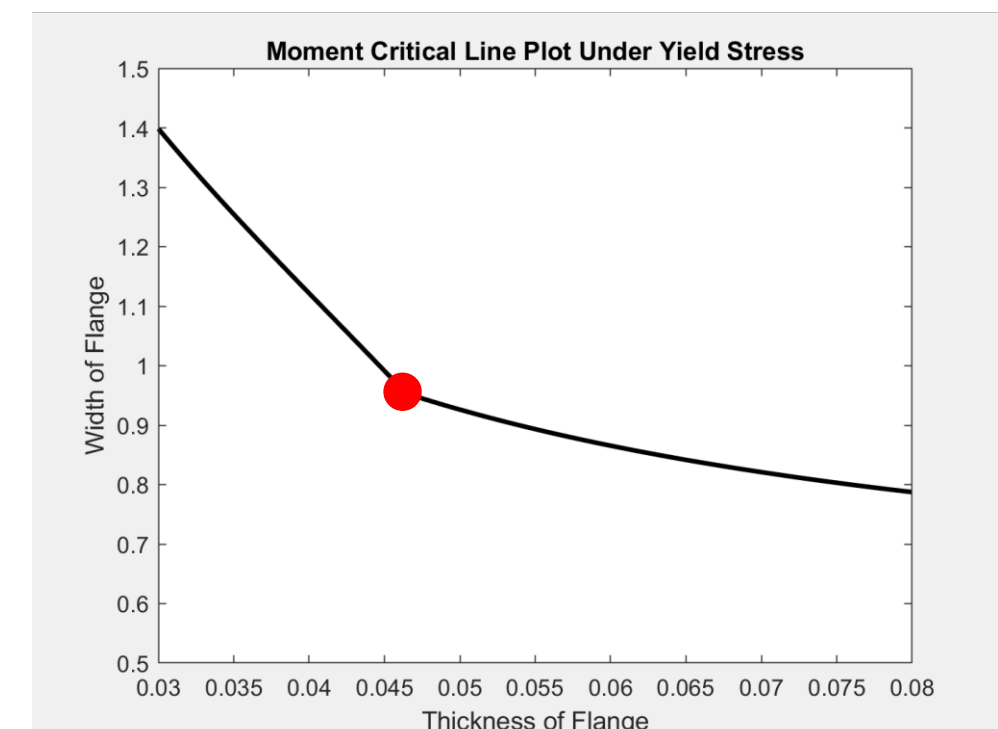
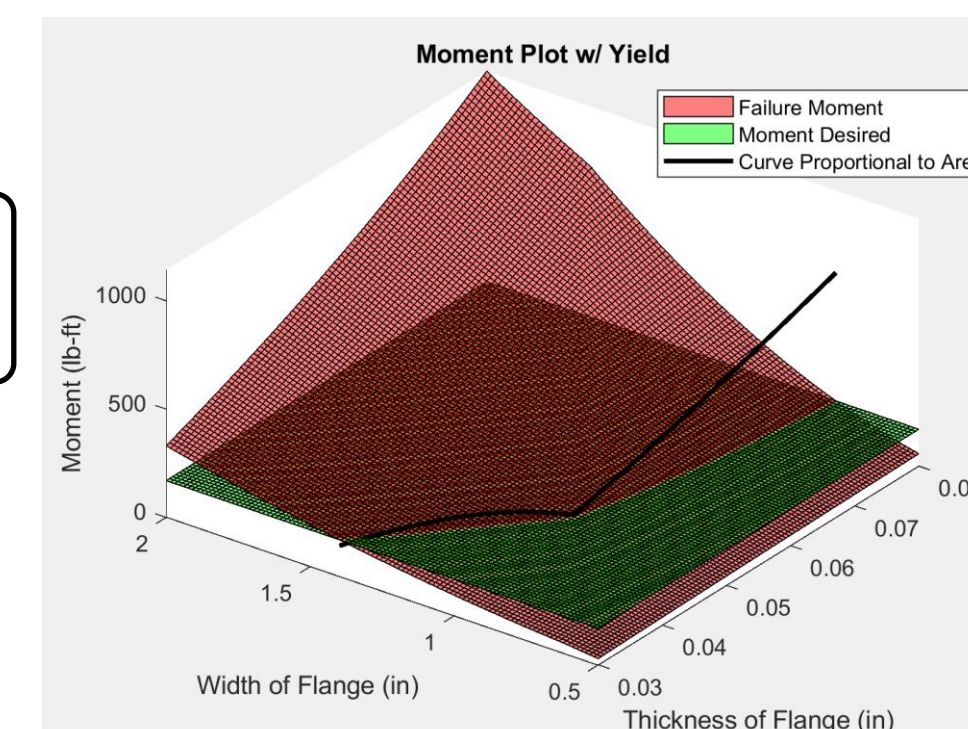
Calculating the maximum bending moment load for certain dimensions of stock may help check an existing structure, but does not help in design, as the dimensions are not known in the first place. Figure 6 shows a surface of the maximum buckling moment for a certain thickness and width and a surface of the desired bending moment. A line proportional to the specific weight for a tube of those dimensions was also plotted to find the point where the weight to strength ratio is minimized.



Failure due to Bending Moment by Buckling (Fig 6)

The problem with this plot is that although it ensures no failure due to buckling, there may still be yielding at the point of maximum compressive stress. To fix this, the maximum stress for each of the dimension combinations on the matrix was calculated and whenever that value exceeded the compressive yield stress, a new maximum moment was assigned, complying with that yield stress. This can be seen on figure 6. At the point of intersection between the two surfaces, a line proportional to weight was also plotted, showing the point of optimized weight to strength ratio. Figure 7 shows the line of critical values that support the desired bending moment, where the optimized value is highlighted in red.

Failure due to Bending Moment (Fig 7 & 8)



### Sources of error:

Sources of error can be determined, which cause the slight discrepancy in the testing results. The testing specimens were manufactured by different companies, which means their modulus of elasticity and yield stress may vary slightly. There is also fillet of different sizes on different tubes, which has not been considered in calculations. Every tube is different and micro-defects will cause failure at different bending moments, even if they have the same specifications. Differences in manufacturing such as autoclave temperature and pressure may also be a factor.

### Conclusions:

The maximum load predictions created considering buckling, tensile, compressive, and shear stress failure were consistent with testing as they showed less than 11% error on the five tests conducted. A lumped mass model successfully reflected the maximum loads throughout the structure of the aircraft. Governing formulas, along with testing results, were used to graphically locate the optimal width and thickness of carbon fiber square tubes used as the main structure of UAS aircraft, which can be of use if the carbon fiber stock is purchased or manufactured in house. The methodology developed can now be used as a fast and accurate way to design the fuselage structure of UAS aircraft. The methodology can also be adapted for use with other materials, shape of stock, and systems, as long as beam theory can be applied.