Modeling and Control of Rocket Ascent in a Vertical Plane

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System Description

Nonlinear differential equations:

$$m\ddot{x} = F_T\cos\delta\sin\theta + \frac{1}{2}C_L\rho v^2 A\sin\alpha\sin\theta - \frac{1}{2}C_D\rho v^2 A\cos\alpha\sin\theta$$

$$m\ddot{z} = F_T\cos\delta\cos\theta + \frac{1}{2}C_L\rho v^2 A\sin\alpha\cos\theta - \frac{1}{2}C_D\rho v^2 A\cos\alpha\cos\theta - mg$$

$$I\ddot{\theta} = -\dot{I}\dot{\theta} - b\dot{\theta} + \frac{d_1}{2}C_D\rho v^2 A\sin\alpha - \frac{d_2}{2}F_T\sin\delta - \frac{d_1}{2}C_L\rho v^2 A\cos\alpha$$

Linearized system:

$$\ddot{x} = \left(\frac{F_{T_0}}{m} - \frac{\frac{1}{2}C_D\rho A v_0^2}{m}\right)\theta$$

$$\Delta\dot{z} = \frac{\Delta F_T}{m} - \frac{C_D\rho A v_0}{m}\Delta\dot{z}$$

$$\ddot{\theta} = -\frac{D_2F_{T_0}}{2I} - \frac{b}{I}\dot{\theta}$$

where:

$$\delta\dot{z} \triangleq \dot{z} - v_0$$

$$\delta F_T \triangleq F_T - F_{T_0}$$

Linear transfer functions:

$$H_{\dot{X}\Theta} = \frac{\dot{X}}{\Theta} = \left(\frac{F_{T_0}}{m} - \frac{\frac{1}{2}C_d\rho A v_0^2}{m}\right)\frac{1}{s} = \frac{g}{s}$$

$$H_{\Delta\dot{Z}\Delta F_T} = \frac{\Delta\dot{Z}}{\Delta F_T} = \frac{\frac{1}{m}}{s + \frac{C_D\rho A v_0}{m}}$$

$$H_{\Theta\Delta} = \frac{\Theta}{\Delta} = -\frac{d_2F_{t_0}}{2I}\left(\frac{1}{s\left(s+\frac{b}{I}\right)}\right)$$

Linearization points:

$\theta = 0$
$\dot{z}$ constant
$\dot{x} = 0$

Linearization Assumptions (near-vertical flight):

$\alpha \approx 0$
$C_L \approx 0$
$v \approx \dot{z}$
δ small

Linearization Assumptions (simplification):

m remains constant (unrealistic)
$\dot{I} = 0$ if constant m

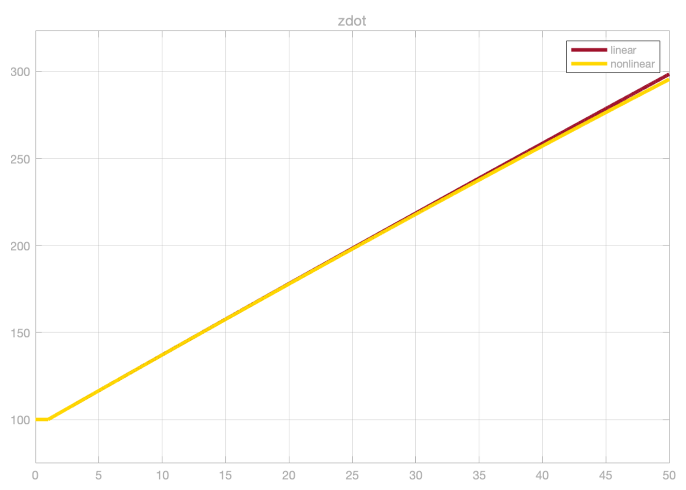
System Behavior and Considerations

Controller Purpose:

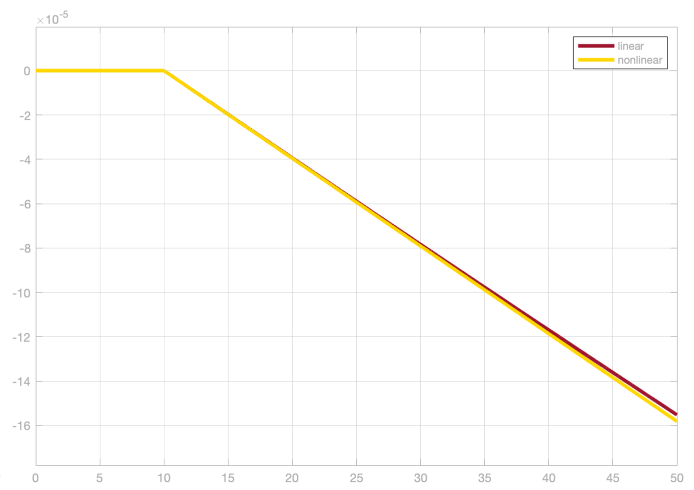
Control F_T to achieve desired $\dot{z}$
Control δ to achieve desired θ
If both of these are held constant, considered cruise control

Linear and Nonlinear Agreement:

Very accurate even over the course of seconds / tens of seconds

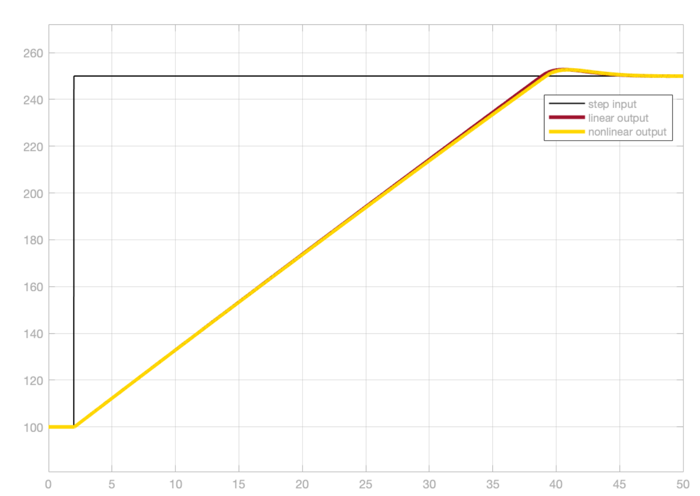


Response of $\dot{z}$ to step F_T input



Response of θ to brief "impulse" δ input

Application of Linearly-Designed Controllers to Nonlinear Model



Response of $\dot{z}$ to reach desired velocity

Response of θ to output disturbance

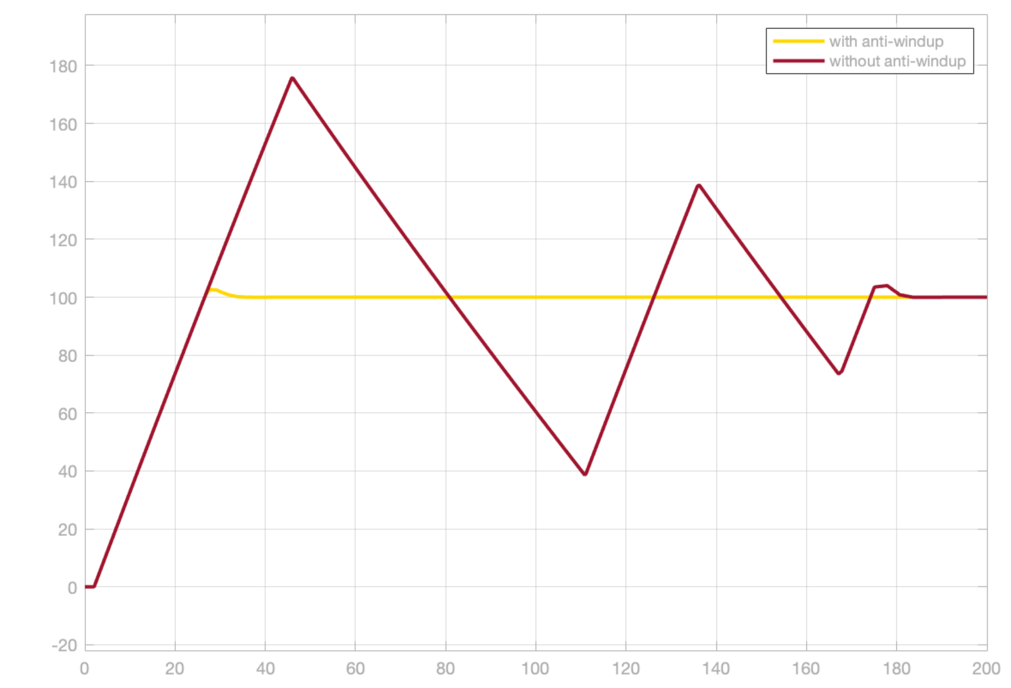
Note:
Approximate values for parameters are taken from the SpaceX Falcon 9 rocket, according to the specifications at https://www.spacex.com/media/Falcon_Users_Guide_082020.pdf

Saturation / Anti-Windup:

Fixed thrust range during ascent
Thrust range limited by throttle capabilities of engines

The Need for Anti-Windup:

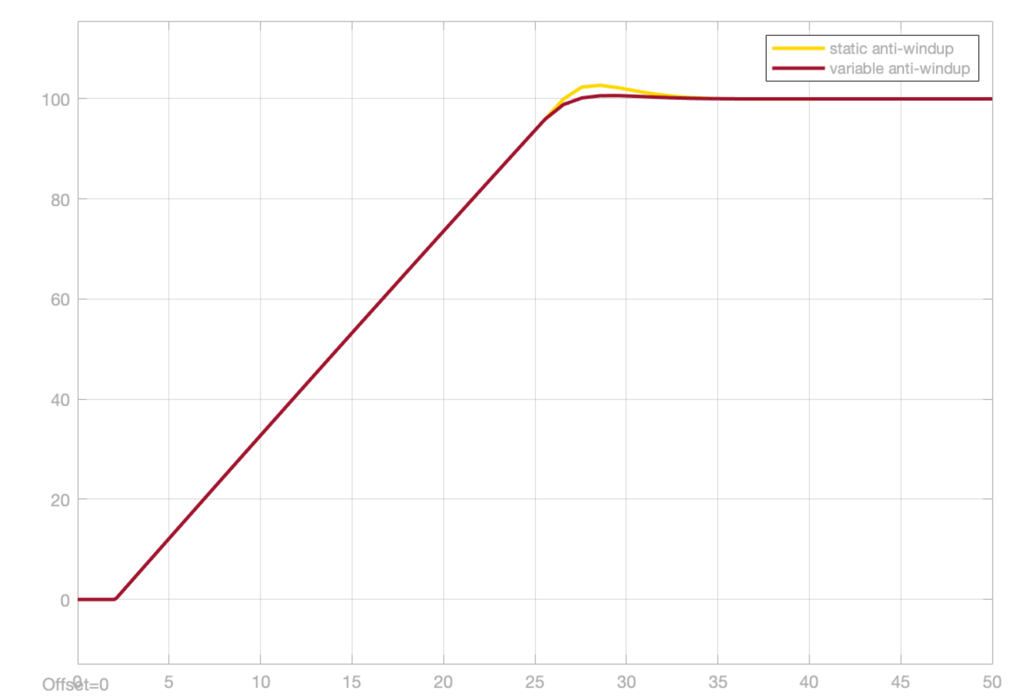
Accumulated error in controller integrator despite actuator saturating
Causes large overshoots and undershoots



Controlled $\dot{z}$ with and without anti-windup

Improved Anti-Windup:

Limits take into account proportional part of controller
Lessens overshoot even more because there is no saturation unaccounted for

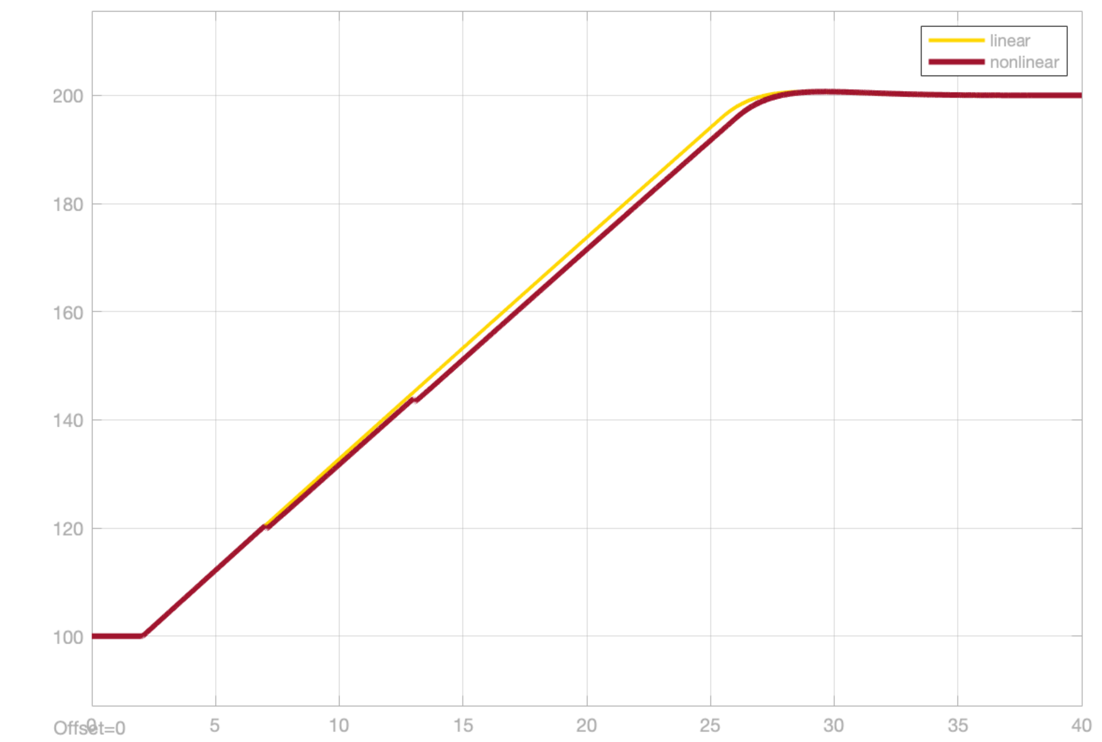


Controlled $\dot{z}$ with static and variable anti-windup

Results

Coupling effects:

Nonlinear system has coupling between variables
These are assumed insignificant for linearization
Still noticeable, but small enough not to matter during control design process
In particular, $\dot{z}$ decreases when θ is nonzero



Response of controlled $\dot{z}$ to perturbations in θ

Takeaways:

Variable anti-windup prevents overshoot substantially
In vertical flight, linear model approximates nonlinear model very well
Adjusting parameters of controller based on operating point seemed to decrease performance (not discussed)

Next Steps:

Test accuracy of linearization at nonzero values of θ
Study effects of variable mass
Expand to 3 spacial dimensions, 9 DOF
Control for position (x and z) instead of cruise control
Examine how system changes for rocket during descent
Design sequence for controlled descent
Design controllers for controlled descent

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