A Novel Approach to Perform Rank-one Updates for Machine Learning

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Introduction

- Computational errors encountered in Machine Learning (ML) can lead these algorithms astray.
- Roundoff errors due to fixed-precision computations have been given little attention.
- For the most part, these errors are associated with solving System of Linear Equations (SLE).
- Transforming ML algorithms into unlimited precision is computationally expensive.
- Rank-one updates provide computational savings by taking advantage that the SLEs solved are close to each other.
- There is a need to perform these updates without any errors.

Objective

The goal is to develop an algorithm which performs Rank-one updates which is not only computationally fast but is also free from rounding errors.

Idea

- The Round-off Error Free (REF) framework is founded on the Integer Preserving Gaussian Elimination algorithm.
- This framework is used to solve rational SLEs using integer arithmetic without any rounding errors.
- In this framework, there is a REF LU factorization that can be efficiently computed to solve SLEs.
- Our main idea is to update the factorization coefficients for the types of changes that are encountered in ML algorithms.

Methodology and Results

- Building the REF-LU factorization from scratch takes $O(n^3)$ operations; hence, we focus on developing an update algorithm that takes only $O(n^2)$ operations without any rounding errors.
- Let A be a $(n \times n)$nonsingular matrix whose factorization is known, the Rank-one update of A is defined as,

REF-LU(A) → *Rank-one Update* → **REF-LU(B)**

$$B = A + \alpha XX^t$$

α is an integer scalar

X is a column vector with n entries

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} + \alpha \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} [x_1 \dots x_n]$$

Example

$$\underset{A}{\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}} + \underset{\alpha}{2} \underset{X}{\begin{bmatrix} 1 \\ -1 \end{bmatrix}} \underset{X^t}{\begin{bmatrix} 1 & -1 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 & 1 \\ 2 & 7 \end{bmatrix}}$$

Computational Results

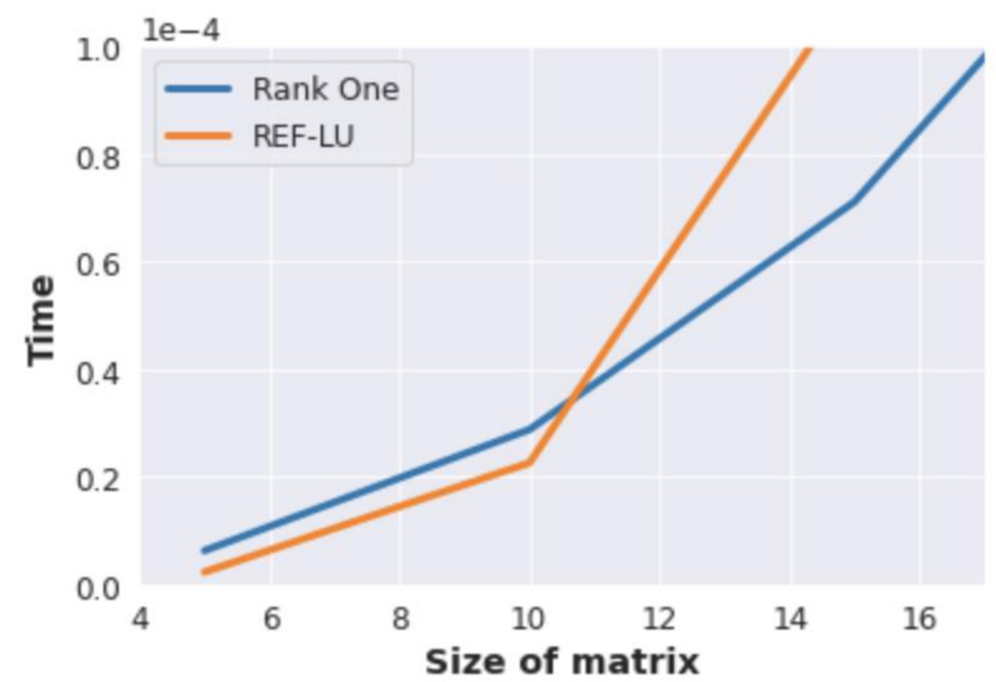
Size of Matrix	Time Taken in seconds	
	REF-LU	Rank-one Update
5	2.26e-6	6.32e-6
10	2.27e-5	2.87e-5
50	0.0041	0.0012
100	0.0472	0.0069
600	40.56	0.8929

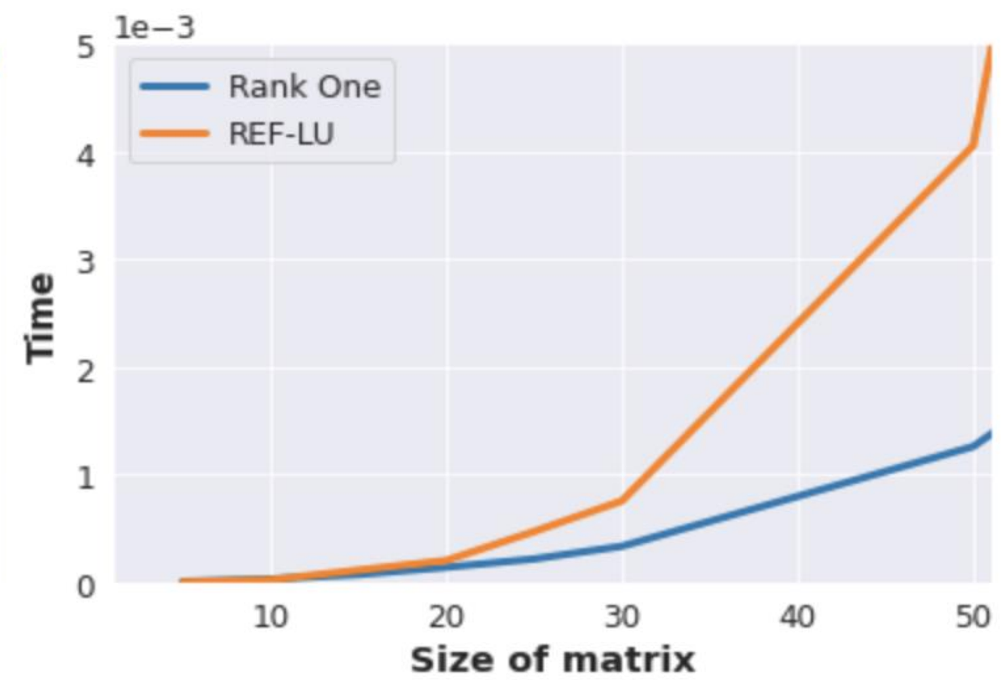
We have devised an update algorithm that has been tested numerically.

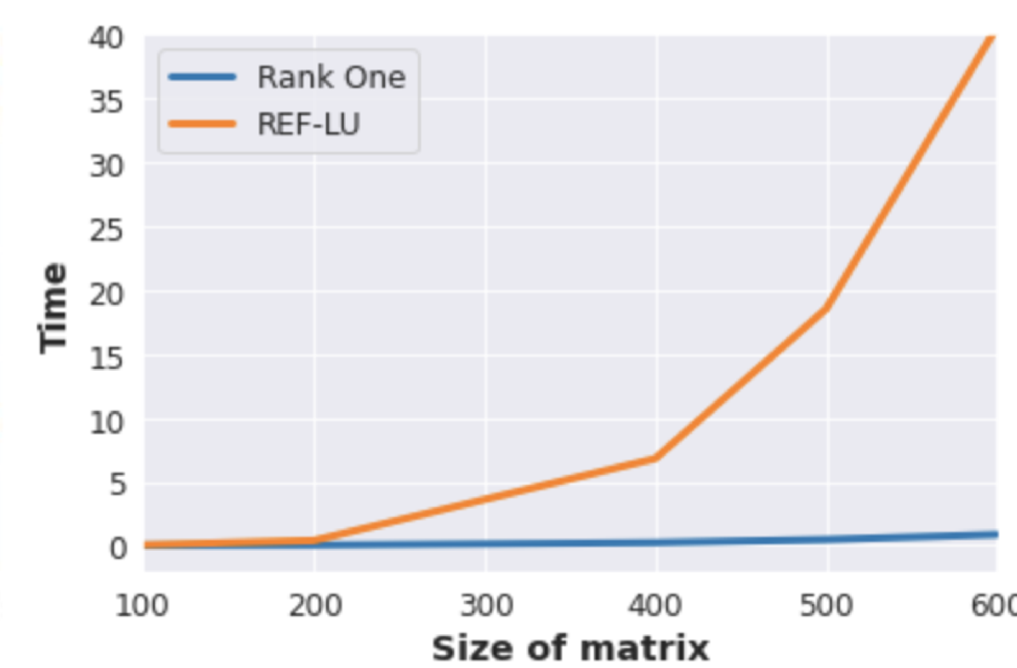
Highlight #1 Initially when the size of the matrix (N) is small (less than 10), there is not much computational difference between our method and factorizing B from scratch.

Highlight #2 When size increases, the time taken to perform the factorization from scratch increases very fast.

Highlight #3 When N=600, Rank-one Update is 40 times faster.







Implementation Details

- The elements of the matrix were selected randomly from the numbers -10 to +10.
- To perform high precision operations, the algorithm was programmed in C++ using the GNU GMP library .
- A 1.4 GHz Quad core CPU with 16GB memory was used to perform the computations.

Future Work

- Do a formal derivation of the algorithm theoretically.
- Code other algorithms and compare the results.
- Use a benchmark dataset or generate a synthetic dataset from Machine learning domain to test the algorithm.
- Generalize this approach to perform other low rank updates.

References

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